

1/M Corrections to Baryonic Form Factors in the Quark Model

Hai-Yang Cheng and B. Tseng

Institute of Physics, Academia Sinica
Taipei, Taiwan 11529, Republic of China

Abstract

Weak current-induced baryonic form factors at zero recoil are evaluated in the rest frame of the heavy parent baryon using the nonrelativistic quark model. Contrary to previous similar work in the literature, our quark model results do satisfy the constraints imposed by heavy quark symmetry for heavy-heavy baryon transitions at the symmetric point $v \cdot v' = 1$ and are in agreement with the predictions of the heavy quark effective theory for antitriplet-antitriplet heavy baryon form factors at zero recoil evaluated to order $1/m_Q$. Furthermore, the quark model approach has the merit that it is applicable to any heavy-heavy and heavy-light baryonic transitions at maximum q^2 . Assuming a dipole q^2 behavior, we have applied the quark model form factors to nonleptonic, semileptonic and weak radiative decays of the heavy baryons. It is emphasized that the flavor suppression factor occurring in many heavy-light baryonic transitions, which is unfortunately overlooked in most literature, is very crucial towards an agreement between theory and experiment for the semileptonic decay $\Lambda_c \rightarrow \Lambda e^+ \nu_e$. Predictions for the decay modes $\Lambda_b \rightarrow J/\psi \Lambda$, $\Lambda_c \rightarrow p \phi$, $\Lambda_b \rightarrow \Lambda \gamma$, $\Xi_b \rightarrow \Xi \gamma$, and for the semileptonic decays of Λ_b , $\Xi_{b,c}$ and Ω_b are presented.

I. Introduction

In the heavy quark effective theory (HQET), there are two different types of $1/m_Q$ corrections to the hadronic form factors: one from the $1/m_Q$ correction to the current operators, and the other from the presence of higher dimensional operators in the effective Lagrangian [1]. The latter amounts to the hadronic wave-function modifications. In general, the predictive power of HQET for $1/m_Q$ effects is very limited by the fact that we do not know how to carry out first-principles calculations for the hadronic matrix elements in which higher dimensional kinetic and chromo-magnetic operators O_1 and O_2 are inserted. Consequently, several new unknown functions are necessarily introduced besides the leading Isgur-Wise functions. For example, to order Λ_{QCD}/m_c , there are four new subleading Isgur-Wise functions $\eta(\omega)$, $\chi_1(\omega)$, $\chi_2(\omega)$ and $\chi_3(\omega)$ ¹ for $B \rightarrow D$ transition, whose normalizations are not determined except that χ_1 and χ_3 vanish at the zero-recoil point $\omega \equiv v \cdot v' = 1$ [2]. Since the Isgur-Wise functions are not calculable from perturbative QCD or HQET, a calculation of them should be resorted to some models. It is known that the Isgur-Wise functions have some simple expressions in the quark model. Denoting the heavy meson wave function by

$$\psi = \psi_0 + \psi_{\text{kin}} + \psi_{\text{mag}} + \cdots, \quad (1)$$

where ψ_0 is the wave function in the heavy quark limit, ψ_{kin} and ψ_{mag} are the $1/m_Q$ corrections to the wave function due to the operators O_1 and O_2 respectively, the Isgur-Wise function $\xi(v \cdot v')$ simply measures the degree of overlap between the wave functions $\psi_0(v)$ and $\psi_0(v')$, while χ_1 (χ_3) can be expressed as the overlap integral of ψ_{kin} (ψ_{mag}) and ψ_0 [3].

In the heavy baryon case, there exist three baryonic Isgur-Wise functions in the heavy quark limit: $\zeta(\omega)$ for antitriplet-antitriplet transition, and $\xi_1(\omega)$, $\xi_2(\omega)$ for sextet-sextet transition. In principle, these functions are also calculable in the quark model though they are more complicated. However, a tremendous simplification occurs in the antitriplet-antitriplet heavy baryon transition, e.g. $\Lambda_b \rightarrow \Lambda_c$: $1/m_Q$ corrections only amount to renormalizing the function $\zeta(\omega)$ and no further new function is needed [4]. This simplification stems from the fact that the chromo-magnetic operator does not contribute to $\Lambda_b \rightarrow \Lambda_c$ and that the diquark of the antitriplet heavy baryon is a spin singlet. Therefore, $1/m_b$ and $1/m_c$ corrections to $\Lambda_b \rightarrow \Lambda_c$ and $\Xi_b \rightarrow \Xi_c$ form factors are predictable in HQET and certain heavy quark symmetry relations among baryonic form factors remain intact. Since HQET is a theory, its prediction is model independent.

Going beyond the antitriplet-antitriplet heavy baryon transition, the predictive power of HQET for form factors at order $1/m_Q$ is lost owing to the fact that $1/m_Q$ corrections

¹We follow the notation of Ref.[1] for subleading Isgur-Wise functions. The new function $\eta(\omega)$ arises from the matrix element of the $1/m_Q$ -expanded current operator.

due to wave function modifications arising from O_1 and especially O_2 are not calculable by perturbative QCD. Therefore, it is appealing to have model calculations which enable us to estimate the $1/m_Q$ corrections for other baryon form factors. To our knowledge, two different quark-model calculations [5,6] are available in the literature. In order to ensure that quark model results are reliable and trustworthy, model predictions, when applied to heavy-heavy baryon transitions, must satisfy all the constraints imposed by heavy quark symmetry. In the heavy quark limit, normalization of the form factors at zero recoil is fixed by heavy quark symmetry. To order $1/m_Q$, antitriplet-antitriplet (i.e. $\Lambda_Q \rightarrow \Lambda_{Q'}$, $\Xi_Q \rightarrow \Xi_{Q'}$) form factors are also calculable in HQET. Unfortunately, none of the calculations presented in [5,6] is in agreement with the predictions of HQET. For example, several heavy quark symmetry relations between baryon form factors are not respected in Ref.[5]. While this discrepancy is resolved in Ref.[6], the $1/m_Q$ corrections obtained in this reference are still inconsistent with HQET in magnitude.

The purpose of the present paper is to demonstrate that the nonrelativistic quark model for baryon form factors, when evaluated properly, does respect all the heavy quark symmetry constraints, including $1/m_Q$ corrections to $\Lambda_Q \rightarrow \Lambda_{Q'}$ ($\Xi_Q \rightarrow \Xi_{Q'}$) form factors predicted by HQET. As a consequence, this model does incorporate the features of heavy quark symmetry and can be used to compute form factors beyond the arena of HQET. Since the quark-model wave function best resembles the hadronic state in the rest frame, we will thus first evaluate form factors at the zero recoil kinetic point. Instead of evaluating the baryonic Isgur-Wise functions, which are beyond the scope of a nonrelativistic quark model, we will make the conventional pole dominance assumption for the q^2 dependence to extrapolate the form factors from maximum q^2 to the desired q^2 point. Since corrections to the form factors due to the modified wave functions vanish at zero recoil (see Sec. II), the nonrelativistic quark model applies equally well to the sextet-sextet heavy baryon transition, e.g. $\Omega_b \rightarrow \Omega_c$ at the symmetric point $v \cdot v' = 1$. Moreover, it becomes meaningful to consider in this model the $1/m_s$ corrections to, for example, $\Lambda_Q \rightarrow \Lambda$ and $\Xi_Q \rightarrow \Xi$ form factors at maximum q^2 so long as the recoil momentum is smaller than the m_s scale.

The layout of the present paper is as follows. In Sec. II we will derive, within the framework of the nonrelativistic quark model, the $1/m_Q$ and $1/m_q$ corrections (a distinction between m_Q and m_q will be defined in Sec. II), coming from the current operator, to the baryonic form factors at zero recoil. The HQET predictions for $\Lambda_b \rightarrow \Lambda_c$ at order $1/m_Q$ are reproduced in this quark-model calculation. Assuming a pole behavior for the q^2 dependence of the form factors, we will apply in Sec. III the quark-model results for baryonic form factors to nonleptonic weak decays, semileptonic decays and weak radiative decays. Sec. IV comes to our discussion and conclusion.

II. Baryonic Form Factors in the Nonrelativistic Quark Model

The general expression for the baryonic transition $B_i \rightarrow B_f$ reads

$$\begin{aligned} \langle B_f(p_f) | V_\mu - A_\mu | B_i(p_i) \rangle &= \bar{u}_f [f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu \\ &\quad - (g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q_\mu) \gamma_5] u_i, \end{aligned} \quad (2)$$

where $q = p_i - p_f$. When both baryons are heavy, it is also convenient to parametrize the matrix element in terms of the velocities v and v' :

$$\begin{aligned} \langle B_f(v') | V_\mu - A_\mu | B_i(v) \rangle &= \bar{u}_f [F_1(\omega) \gamma_\mu + F_2(\omega) v_\mu + F_3(\omega) v'_\mu \\ &\quad - (G_1(\omega) \gamma_\mu + G_2(\omega) v_\mu + G_3(\omega) v'_\mu) \gamma_5] u_i, \end{aligned} \quad (3)$$

with $\omega \equiv v \cdot v'$. The form factors F_i and G_i are related to f_i and g_i via

$$\begin{aligned} f_1 &= F_1 + \frac{1}{2}(m_i + m_f) \left(\frac{F_2}{m_i} + \frac{F_3}{m_f} \right), \\ f_2 &= \frac{1}{2} \left(\frac{F_2}{m_i} + \frac{F_3}{m_f} \right), \\ f_3 &= \frac{1}{2} \left(\frac{F_2}{m_i} - \frac{F_3}{m_f} \right), \\ g_1 &= G_1 - \frac{1}{2}(m_i - m_f) \left(\frac{G_2}{m_i} + \frac{G_3}{m_f} \right), \\ g_2 &= \frac{1}{2} \left(\frac{G_2}{m_i} + \frac{G_3}{m_f} \right), \\ g_3 &= \frac{1}{2} \left(\frac{G_2}{m_i} - \frac{G_3}{m_f} \right), \end{aligned} \quad (4)$$

where m_i (m_f) is the mass of B_i (B_f). Since the quark model is most trustworthy when the baryon is static, we will thus evaluate the form factors at zero recoil $\vec{q} = 0$ (or $q^2 = (m_i - m_f)^2$) in the rest frame of the parent baryon B_i . Note that in order to determine the form factors $f_{2,3}$ and $g_{2,3}$, we need to keep the small recoil momentum \vec{q} in Eq.(2) when recasting the 4-component Dirac spinors in terms of the 2-component Pauli spinors.

In order to calculate the form factors using the nonrelativistic quark model, we write [5]

$$\begin{aligned} \langle B_f | V_0 | B_i \rangle &= \chi_f^\dagger \tilde{V}_0(q^2) \chi_i, \\ \langle B_f | A_0 | B_i \rangle &= \chi_f^\dagger \vec{\sigma} \cdot \vec{q} \tilde{A}_0(q^2) \chi_i, \\ \langle B_f | \vec{V} | B_i \rangle &= \chi_f^\dagger [\vec{q} \tilde{V}_V(q^2) + i \vec{\sigma} \times \vec{q} \tilde{V}_M(q^2)] \chi_i, \\ \langle B_f | \vec{A} | B_i \rangle &= \chi_f^\dagger [\vec{\sigma} \tilde{A}_S(q^2) + \vec{q} (\vec{\sigma} \cdot \vec{q}) \tilde{A}_T(q^2)] \chi_i, \end{aligned} \quad (5)$$

where χ is a Pauli spinor. In the rest frame of B_i , we find from Eqs.(2) and (5) that the scalar coefficients \tilde{V} and \tilde{A} at maximum q^2 are given by

$$\begin{aligned}
\tilde{V}_0(q_m^2) &= f_1 + \Delta m f_3, \\
\tilde{V}_V(q_m^2) &= \frac{1}{2m_f}(-f_1 + \Delta m f_2) + f_3, \\
\tilde{V}_M(q_m^2) &= \frac{1}{2m_f}[-f_1 + (m_i + m_f)f_2], \\
\tilde{A}_0(q_m^2) &= \frac{1}{2m_f}(-g_1 + \Delta m g_3) + g_2, \\
\tilde{A}_S(q_m^2) &= g_1 + \Delta m g_2, \\
\tilde{A}_T(q_m^2) &= \frac{1}{2m_f}(-g_2 + g_3),
\end{aligned} \tag{6}$$

where $q_m^2 \equiv q_{\max}^2 = (\Delta m)^2$ and $\Delta m = m_i - m_f$. Inverting the above equations gives

$$\begin{aligned}
f_1(q_m^2) &= \left(1 - \frac{\Delta m}{2m_i}\right) \tilde{V}_0 - \frac{\Delta m(m_i + m_f)}{2m_i} \tilde{V}_V + \frac{(\Delta m)^2}{2m_i} \tilde{V}_M, \\
f_2(q_m^2) &= \frac{1}{2m_i} \tilde{V}_0 - \frac{\Delta m}{2m_i} \tilde{V}_V + \frac{m_i + m_f}{2m_i} \tilde{V}_M, \\
f_3(q_m^2) &= \frac{1}{2m_i} \tilde{V}_0 + \frac{m_i + m_f}{2m_i} \tilde{V}_V - \frac{\Delta m}{2m_i} \tilde{V}_M, \\
g_1(q_m^2) &= \left(1 - \frac{\Delta m}{2m_i}\right) \tilde{A}_S - \frac{m_f \Delta m}{m_i} \tilde{A}_0 + \frac{m_f (\Delta m)^2}{m_i} \tilde{A}_T, \\
g_2(q_m^2) &= \frac{1}{2m_i} \tilde{A}_S + \frac{m_f}{m_i} \tilde{A}_0 - \frac{m_f \Delta m}{m_i} \tilde{A}_T, \\
g_3(q_m^2) &= \frac{1}{2m_i} \tilde{A}_S + \frac{m_f}{m_i} \tilde{A}_0 + \frac{m_f(m_i + m_f)}{m_i} \tilde{A}_T.
\end{aligned} \tag{7}$$

Our next task is to employ the nonrelativistic quark model to evaluate the coefficients \tilde{V} and \tilde{A} at $q^2 = q_m^2$. We will follow closely Ref.[6] for this task. Suppose that the parent baryon B_i contains a heavy quark Q and two light quarks q_1 and q_2 behaving as a spectator diquark, and that the final baryon B_f is composed of the quark q (being a heavy quark Q' or a s quark) and the same light diquark as in B_i . Denoting the spatial coordinates of the three quarks in B_i by \vec{r}_Q , \vec{r}_1 and \vec{r}_2 , we define the relative coordinates

$$\vec{R} = \frac{\sum m_j \vec{r}_j}{\tilde{m}_i}, \quad \vec{r}_{12} = \vec{r}_2 - \vec{r}_1, \quad \vec{r}_\ell = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} - \vec{r}_Q, \tag{8}$$

where $\tilde{m}_i = m_Q + m_1 + m_2$, which is in practice close to m_i , so that \vec{r}_{12} is the relative coordinate of the two light quarks, and \vec{r}_ℓ is the relative coordinate of Q and the c.m. of the diquark. It is easily shown that the corresponding relative momenta are [6]

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3,$$

$$\begin{aligned}\vec{p}_{12} &= \frac{m_1}{m_1 + m_2} \vec{p}_2 - \frac{m_2}{m_1 + m_2} \vec{p}_1, \\ \vec{\ell} &= \frac{m_Q}{\tilde{m}_i} (\vec{p}_1 + \vec{p}_2) - \frac{m_1 + m_2}{\tilde{m}_i} \vec{p}_Q.\end{aligned}\tag{9}$$

In the rest frame of the parent baryon, the momenta Q and q are related to the relative momentum $\vec{\ell}$ via

$$\vec{p}_Q = -\vec{\ell}, \quad \vec{p}_q = -\vec{q} - \vec{\ell},\tag{10}$$

and the relative momenta of the quarks in the baryon B_f denoted with primes are related to that in B_i by

$$\vec{p}'_{12} = \vec{p}_{12}, \quad \vec{\ell}' = \vec{\ell} + \frac{m_1 + m_2}{\tilde{m}_f} \vec{q},\tag{11}$$

with $\tilde{m}_f = m_q + m_1 + m_2$. Note that the recoil momentum of the daughter baryon B_f is $-\vec{q}$.

The baryon state is represented in the nonrelativistic quark model by

$$|B_i(\vec{P}, s)\rangle = \int d^3\vec{p}_{12} d^3\vec{\ell} \phi(\vec{p}_{12}, \vec{\ell}) \sum_{s_1, s_2, s_Q} C_{s_1, s_2, s_Q}^s |Q(\vec{p}_Q, s_Q), q_1(\vec{p}_1, s_1), q_2(\vec{p}_2, s_2)\rangle,\tag{12}$$

where C_{s_1, s_2, s_Q}^s is the Clebsch-Gordan coefficient for the combination of three constituent quarks into a spin- $\frac{1}{2}$ baryon with the spin component s along the z direction, and $\phi(\vec{p}_{12}, \vec{\ell})$ is the momentum wave function satisfying the normalization condition:

$$\int d^3\vec{p}_{12} d^3\vec{\ell} |\phi(\vec{p}_{12}, \vec{\ell})|^2 = 1.\tag{13}$$

We shall see that the form factors to be evaluated at zero recoil do not depend on the explicit detail of $\phi(\vec{p}_{12}, \vec{\ell})$. Consider the weak current $J_\mu = \bar{q}\gamma_\mu(1 - \gamma_5)Q$. In the quark model the hadronic matrix element in (2) becomes

$$\begin{aligned}\langle B_f(\vec{P}' = -\vec{q}, s') | J_\mu | B_i(\vec{P} = 0, s) \rangle &= \int d^3\vec{p}'_{12} d^3\vec{\ell}' d^3\vec{p}_{12} d^3\vec{\ell} \\ &\times \delta^3(\vec{p}'_{12} - \vec{p}_{12}) \delta^3\left(\vec{\ell}' - \vec{\ell} - \frac{m_1 + m_2}{\tilde{m}_f} \vec{q}\right) \phi_f(\vec{p}'_{12}, \vec{\ell}') \phi_i(\vec{p}_{12}, \vec{\ell}) \langle s' | J_\mu | s \rangle,\end{aligned}\tag{14}$$

with

$$\langle s' | J_\mu | s \rangle = \sum C_{s_1, s_2, s_q}^{s'} C_{s_1, s_2, s_Q}^s \bar{q}(\vec{p}_q, s_q) \gamma_\mu (1 - \gamma_5) Q(\vec{p}_Q, s_Q).\tag{15}$$

It will become clear shortly that it makes difference to choose $(\vec{p}'_{12}, \vec{\ell}')$ or $(\vec{p}_{12}, \vec{\ell})$ as the

integration variables after integrating over the δ -functions. We thus take the average ²

$$\langle B_f(-\vec{q}, s') | J_\mu | B_i(\vec{0}, s) \rangle = \frac{1}{2} \left[\int d^3 \vec{p}'_{12} d^3 \vec{\ell}' + \int d^3 \vec{p}_{12} d^3 \vec{\ell} \right] \phi_f(\vec{p}'_{12}, \vec{\ell}') \phi_i(\vec{p}_{12}, \vec{\ell}) \langle s' | J_\mu | s \rangle. \quad (16)$$

In the nonrelativistic limit, the Dirac spinors in (15) read

$$\bar{q}(\vec{p}_q, s_q) = \chi^\dagger \left(1, -\frac{\vec{\sigma} \cdot \vec{p}_q}{2m_q} \right), \quad Q(\vec{p}_Q, s_Q) = \left(\frac{1}{\frac{\vec{\sigma} \cdot \vec{p}_Q}{2m_Q}} \right) \chi. \quad (17)$$

Note that $\vec{p}_q = -(\vec{q} + \vec{\ell})$, $\vec{p}_Q = -\vec{\ell}$ when $(\vec{p}_{12}, \vec{\ell})$ are chosen to be the integration variables, and $\vec{p}_q = -(m_q/\tilde{m}_f)\vec{q} - \vec{\ell}$, $\vec{p}_Q = \vec{q}(m_1 + m_2)/\tilde{m}_f - \vec{\ell}$ for the integration variables $(\vec{p}'_{12}, \vec{\ell}')$. Obviously, the integration over $(\vec{p}_{12}, \vec{\ell})$ is in general different from that over $(\vec{p}'_{12}, \vec{\ell}')$.

Substituting (17) into (16) and noting that terms linear in $\vec{\ell}$ ($\vec{\ell}'$) make no contribution after integrating over $\vec{\ell}$ ($\vec{\ell}'$), we find after some manipulation that the scalar coefficients \tilde{V} and \tilde{A} evaluated at $\vec{q} = 0$ are

$$\begin{aligned} \tilde{V}_0(q_m^2)/N_{fi} &= 1, \\ \tilde{V}_V(q_m^2)/N_{fi} &= -\frac{1}{2m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) + \frac{\bar{\Lambda}}{4m_f m_Q}, \\ \tilde{V}_M(q_m^2)/N'_{fi} &= -\frac{1}{2m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) - \frac{\bar{\Lambda}}{4m_f m_Q}, \\ \tilde{A}_0(q_m^2)/N'_{fi} &= -\frac{1}{2m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f} \right) + \frac{\bar{\Lambda}}{4m_f m_Q}, \\ \tilde{A}_S(q_m^2)/N'_{fi} &= 1, \\ \tilde{A}_T(q_m^2)/N'_{fi} &= -\frac{\bar{\Lambda}}{4m_f^2 m_Q}, \end{aligned} \quad (18)$$

where use of the approximation $\tilde{m}_f \approx m_f$ has been made, $\bar{\Lambda} \equiv m_f - m_q$, and

$$N_{fi} = \text{flavor-spin} \langle B_f | b_q^\dagger b_Q | B_i \rangle_{\text{flavor-spin}}, \quad N'_{fi} = \text{flavor-spin} \langle B_f | b_q^\dagger b_Q \sigma_z^Q | B_i \rangle_{\text{flavor-spin}}, \quad (19)$$

with σ_Q acting on the heavy quark Q . In deriving Eq.(18) we have applied the normalization condition (13) for the momentum wave function by assuming flavor independence, $\phi_f = \phi_i$. Since

$$\langle \chi_s | \sigma_z^3 | \chi_s \rangle = -\frac{1}{3}, \quad \langle \chi_A | \sigma_z^3 | \chi_A \rangle = 1, \quad (20)$$

²The analogous expression obtained by Singleton [6] is

$$\langle B_f(-\vec{q}, s') | J_\mu | B_i(\vec{0}, s) \rangle = \left(\frac{m_q}{\tilde{m}_f} \right)^3 \int d^3 \vec{p}_{12} d^3 \vec{\ell} \phi_f(\vec{p}_{12}, \vec{\ell}) \phi_i(\vec{p}_{12}, \vec{\ell}) \langle s' | J_\mu | s \rangle.$$

He noticed that there would be a factor of $(m_Q/\tilde{m}_i)^3$ instead if $(\vec{p}'_{12}, \vec{\ell}')$ were integrated over [these factors do not appear in our Eq.(14) or (16)]. It is argued in [6] that these factors can be neglected since in the spectator model the diquark mass should not affect the rate.

where $\chi_s = (2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)/\sqrt{6}$ is the spin wave function for the sextet heavy baryon and $\chi_A = (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)/\sqrt{2}$ for the antitriplet heavy baryon, it is clear that [6]

$$\eta \equiv \frac{N'_{fi}}{N_{fi}} = \begin{cases} 1 & \text{for antitriplet baryon } B_i, \\ -\frac{1}{3} & \text{for sextet baryon } B_i. \end{cases} \quad (21)$$

It follows from Eqs.(18) and (7) that the form factors at zero recoil are given by

$$\begin{aligned} f_1(q_m^2)/N_{fi} &= 1 - \frac{\Delta m}{2m_i} + \frac{\Delta m}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f}\right) (m_i + m_f - \eta\Delta m) \\ &\quad - \frac{\Delta m}{8m_i m_f} \frac{\bar{\Lambda}}{m_Q} (m_i + m_f + \eta\Delta m), \\ f_2(q_m^2)/N_{fi} &= \frac{1}{2m_i} + \frac{1}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f}\right) [\Delta m - (m_i + m_f)\eta] \\ &\quad - \frac{\bar{\Lambda}}{8m_i m_f m_Q} [\Delta m + (m_i + m_f)\eta], \\ f_3(q_m^2)/N_{fi} &= \frac{1}{2m_i} - \frac{1}{4m_i m_q} \left(1 - \frac{\bar{\Lambda}}{2m_f}\right) (m_i + m_f - \eta\Delta m) \\ &\quad + \frac{\bar{\Lambda}}{8m_i m_f m_Q} (m_i + m_f + \eta\Delta m), \\ g_1(q_m^2)/N_{fi} &= \eta + \frac{\Delta m \bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q}\right) \eta, \\ g_2(q_m^2)/N_{fi} &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q}\right) \eta, \\ g_3(q_m^2)/N_{fi} &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} + \frac{1}{m_f m_Q}\right) \eta, \end{aligned} \quad (22)$$

with $\bar{\Lambda} = m_f - m_q$. When both baryons are heavy, the form factors defined in Eq.(3) have the following expressions at $\omega = 1$:

$$\begin{aligned} F_1(1)/N_{fi} &= \left[1 + \frac{\bar{\Lambda}}{2} \left(\frac{1}{m_q} + \frac{1}{m_Q}\right)\right] \eta, \\ F_2(1)/N_{fi} &= \frac{1}{2}(1 - \eta) - \frac{\bar{\Lambda}}{2m_q} + \frac{\bar{\Lambda}}{4} \left(\frac{1}{m_q} - \frac{1}{m_Q}\right) (1 - \eta), \\ F_3(1)/N_{fi} &= \frac{1}{2}(1 - \eta) - \frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{4} \left(\frac{1}{m_q} + \frac{1}{m_Q}\right) (1 - \eta), \\ G_1(1)/N_{fi} &= \eta, \quad G_2(1)/N_{fi} = -\frac{\bar{\Lambda}}{2m_q} \eta, \quad G_3(1)/N_{fi} = \frac{\bar{\Lambda}}{2m_Q} \eta, \end{aligned} \quad (23)$$

obtained from Eqs.(4) and (22). For antitriplet $\Lambda_b \rightarrow \Lambda_c$ or $\Xi_b \rightarrow \Xi_c$ transition, $N_{fi} = 1$, $\eta = 1$, so we have

$$F_1^{\Lambda_b \Lambda_c}(1) = 1 + \frac{\bar{\Lambda}}{2} \left(\frac{1}{m_c} + \frac{1}{m_b}\right), \quad G_1^{\Lambda_b \Lambda_c}(1) = 1,$$

$$\begin{aligned}
F_2^{\Lambda_b \Lambda_c}(1) &= G_2^{\Lambda_b \Lambda_c}(1) = -\frac{\bar{\Lambda}}{2m_c}, \\
F_3^{\Lambda_b \Lambda_c}(1) &= -G_3^{\Lambda_b \Lambda_c}(1) = -\frac{\bar{\Lambda}}{2m_b},
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
f_1^{\Lambda_b \Lambda_c}(q_m^2) = g_1^{\Lambda_b \Lambda_c}(q_m^2) &= 1 + \frac{\Delta m \bar{\Lambda}}{4} \left(\frac{1}{m_{\Lambda_b} m_c} - \frac{1}{m_{\Lambda_c} m_b} \right), \\
f_2^{\Lambda_b \Lambda_c}(q_m^2) = g_3^{\Lambda_b \Lambda_c}(q_m^2) &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_{\Lambda_b} m_c} + \frac{1}{m_{\Lambda_c} m_b} \right), \\
f_3^{\Lambda_b \Lambda_c}(q_m^2) = g_2^{\Lambda_b \Lambda_c}(q_m^2) &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_{\Lambda_b} m_c} - \frac{1}{m_{\Lambda_c} m_b} \right),
\end{aligned} \tag{25}$$

and similar expressions for $\Xi_b \rightarrow \Xi_c$. Therefore, there is only one independent $\Lambda_b \rightarrow \Lambda_c$ form factor in the heavy quark limit. The relevant HQET predictions to the zeroth order of α_s are [4]

$$\begin{aligned}
F_1^{\Lambda_b \Lambda_c}(\omega) &= 1 + \frac{\bar{\Lambda}}{2} \left(\frac{1}{m_c} + \frac{1}{m_b} \right), \\
F_2^{\Lambda_b \Lambda_c}(\omega) &= G_2^{\Lambda_b \Lambda_c}(\omega) = -\frac{\bar{\Lambda}}{m_c} \frac{1}{1 + \omega}, \\
F_3^{\Lambda_b \Lambda_c}(\omega) &= -G_3^{\Lambda_b \Lambda_c}(\omega) = -\frac{\bar{\Lambda}}{2m_b} \frac{1}{1 + \omega}, \\
G_1^{\Lambda_b \Lambda_c}(\omega) &= 1 - \frac{\bar{\Lambda}}{2} \left(\frac{1}{m_c} + \frac{1}{m_b} \right) \frac{1 - \omega}{1 + \omega}.
\end{aligned} \tag{26}$$

We see that the nonrelativistic quark model predictions for $\Lambda_b \rightarrow \Lambda_c$ form factors at the symmetric point $\omega = 1$ are in agreement with HQET up to the order of $1/m_b$ and $1/m_c$, as it should be.

In the heavy quark limit, the sextet $\Sigma_b \rightarrow \Sigma_c$ or $\Omega_b \rightarrow \Omega_c$ transition at $v \cdot v' = 1$ is predicted by HQET to be (see e.g. Ref.[1])

$$\langle \Sigma_c(v', s') | V_\mu - A_\mu | \Sigma_b(v, s) \rangle = \bar{u}_{\Sigma_c}(v', s') [\gamma_\mu (1 + \gamma_5) - 2(v + v')_\mu] u_{\Sigma_b}(v, s). \tag{27}$$

This leads to the sextet-sextet baryonic form factors

$$F_1^{\Sigma_b \Sigma_c}(1) = G_1^{\Sigma_b \Sigma_c}(1) = -\frac{1}{3}, \quad F_2^{\Sigma_b \Sigma_c}(1) = F_3^{\Sigma_b \Sigma_c}(1) = \frac{2}{3}, \quad G_2^{\Sigma_b \Sigma_c}(1) = G_3^{\Sigma_b \Sigma_c}(1) = 0, \tag{28}$$

and

$$f_1^{\Sigma_b \Sigma_c}(q_m^2) = -\frac{1}{3} \left[1 - (m_{\Sigma_b} + m_{\Sigma_c}) \left(\frac{1}{m_{\Sigma_b}} + \frac{1}{m_{\Sigma_c}} \right) \right],$$

$$\begin{aligned}
f_2^{\Sigma_b \Sigma_c}(q_m^2) &= \frac{1}{3} \left(\frac{1}{m_{\Sigma_b}} + \frac{1}{m_{\Sigma_c}} \right), \\
f_3^{\Sigma_b \Sigma_c}(q_m^2) &= \frac{1}{3} \left(\frac{1}{m_{\Sigma_b}} - \frac{1}{m_{\Sigma_c}} \right), \\
g_1^{\Sigma_b \Sigma_c}(q_m^2) &= -\frac{1}{3}, \quad g_2^{\Sigma_b \Sigma_c}(q_m^2) = g_3^{\Sigma_b \Sigma_c}(q_m^2) = 0,
\end{aligned} \tag{29}$$

and similar results for $\Omega_b \rightarrow \Omega_c$. Since $N_{fi} = 1$ and $\eta = -1/3$ for sextet-sextet heavy baryon transitions, it is evident from Eqs.(22) and (23) that our quark model results for $\Sigma_b \rightarrow \Sigma_c$ and $\Omega_b \rightarrow \Omega_c$ form factors evaluated at zero recoil in the heavy quark limit are in agreement with the constraints imposed by heavy quark symmetry.

We now make a comparison with the quark model calculations in Refs.[5,6]. Quark- and bag-model wave functions in the coordinate space are used to evaluate the baryonic form factors by Pérez-Marcial *et al.* [5]. However, their results (11a-11f) at zero recoil,³ when applied to $\Lambda_b \rightarrow \Lambda_c$ and $\Sigma_b \rightarrow \Sigma_c$, are in disagreement with HQET. In fact, the heavy quark symmetry relations $f_1 = g_1$, $f_2 = g_3$ and $f_3 = g_2$ for $\Lambda_b \rightarrow \Lambda_c$ transition and $g_1 = -1/3$, $g_2 = g_3 = 0$ for $\Sigma_b \rightarrow \Sigma_c$ implied by HQET are not respected by Ref.[5]. Moreover, the dimensionless $\Lambda_b \rightarrow \Lambda_c$ form factors $m_{\Lambda_b} f_{2,3}$ and $m_{\Lambda_b} g_{2,3}$ vanish in the heavy quark limit according to HQET. The reader can check that the form factors obtained in Ref.[5] do not satisfy this feature of heavy quark symmetry.

Our evaluation of baryonic form factors is quite close to that of Singleton [6] except mainly for Eq.(16), in which we have taken the average of the integrations over $(\vec{p}_{12}, \vec{\ell})$ and $(\vec{p}'_{12}, \vec{\ell}')$ (see the footnote there). Besides the $1/m_q$ corrections we have also included $1/m_Q$ effects, which are not taken into account in [6]. Recasting (3.48)-(3.51) of Ref.[6] into the form factors used here gives

$$\begin{aligned}
f_1^{\Lambda_b \Lambda_c}(q_m^2) &= g_1^{\Lambda_b \Lambda_c}(q_m^2) = 1 + \frac{\Delta m}{2m_{\Lambda_b}} \frac{\bar{\Lambda}}{m_c}, \\
f_2^{\Lambda_b \Lambda_c}(q_m^2) &= f_3^{\Lambda_b \Lambda_c}(q_m^2) = g_2^{\Lambda_b \Lambda_c}(q_m^2) = g_3^{\Lambda_b \Lambda_c}(q_m^2) = -\frac{\bar{\Lambda}}{2m_c} \frac{1}{m_{\Lambda_b}},
\end{aligned} \tag{30}$$

for $\Lambda_b \rightarrow \Lambda_c$ transition. Comparing with (25), it is evident that the $1/m_c$ corrections in (30) are too large by a factor of 2; that is, the quark model calculations by Singleton do satisfy the aforementioned heavy quark symmetry relations, but are still not consistent with HQET in magnitude.

The $B_i \rightarrow B_f$ baryonic form factors (22) at maximum q^2 obtained in the nonrelativistic quark model are the main results in the present paper. The $1/m_Q$ and $1/m_q$ effects in

³The parameters α_1 and α_2 defined in Eq.(12) of Ref.[5] correspond to our N_{fi} and N'_{fi} , respectively.

(22) arise from the modification to the current operator. Although as far as $\Lambda_Q \rightarrow \Lambda_{Q'}$ and $\Xi_Q \rightarrow \Xi_{Q'}$ are concerned, the nonrelativistic quark model predictions for the form factors at $\omega = 1$ are in accordance with HQET, the two approaches differ in two main aspects: (i) Unlike the nonrelativistic quark model, HQET provides a systematic Λ_{QCD}/m_Q expansion, which can be treated perturbatively if $m_Q \gg \Lambda_{\text{QCD}}$. Near zero recoil in the rest frame of the parent baryon, the quark model result for $1/m_q$ corrections is trustworthy since $|\vec{q}|/m_q \ll 1$, where $-\vec{q}$ is the recoil momentum of the daughter baryon. Consequently, contrary to HQET, $1/m_s$ modifications to the form factors near $v \cdot v' = 1$ become meaningful in the quark model. (ii) Going beyond the antitriplet-antitriplet heavy baryon transition, HQET loses its predictive power for form factors at order $1/m_Q$ since $1/m_Q$ corrections due to wave function modifications arising from O_1 and O_2 are not calculable by perturbative QCD. However, such corrections are expected to vanish at zero recoil in the quark model.

⁴ This is so because the physical results at the symmetric point $\omega = 1$, where both parent and daughter baryons are at rest, should be independent of the explicit form of the wave function. Modifications to the wave function come from the operators O_1 and O_2 acting on ψ_0 [see Eq.(1)], whose explicit expression is model dependent. As a result, the nonrelativistic quark model results (22) for weak current-induced form factors evaluated at maximum q^2 are applicable to any heavy-heavy and heavy-light baryonic transitions.

At this point, we would like to examine the underlying assumptions we have made during the course of deriving (18) or (22). The assumptions are (i) the approximation of the weak binding mass $\tilde{m}_f (= m_q + m_1 + m_2)$ with the mass m_f of the daughter baryon, (ii) flavor independence of the momentum wave function, $\phi_f = \phi_i$, and (iii) the average of two momenta integrals in (16). Assumption (i) is justified by the fact that \tilde{m}_f and m_f for charmed and octet baryons are very close for $m_u = 338$ MeV, $m_d = 322$ MeV, $m_s = 510$ MeV (see p.1729 of Ref.[7]) and $m_c = 1.6$ GeV. In contrast, assumptions (ii) and (iii) are less solid. First, the momentum wave function ϕ is truly flavor independent only in the heavy quark limit. Second, a simple average of two different momenta integrals taken in (16) seems somewhat arbitrary. Nevertheless, the present prescription works empirically as it does agree with HQET at order $1/m_Q$. Since the hadronic matrix element (14) should in principle be independent of the integration order, this probably means that flavor dependence of ϕ , which is of order $1/m_Q$ and $1/m_q$ is compensated by similar effects in (iii). A full understanding of the empirical agreement between present approach and HQET needs to be pursued.

Experimentally, the only information available so far is the form-factor ratio measured

⁴This is known to be true in the meson case. Among the four subleading Isgur-Wise functions $\eta(\omega)$, $\chi_{1,2,3}(\omega)$ (see the Introduction), we know that χ_1 and χ_3 vanish at $\omega = 1$ and that $\eta(\omega) = \chi_2(\omega) = 0$ in the quark model [3].

in the semileptonic decay $\Lambda_c \rightarrow \Lambda e \bar{\nu}$. In the heavy c -quark limit, there are two independent form factors in $\Lambda_c \rightarrow \Lambda$ transition [8]

$$\langle \Lambda(p) | \bar{s} \gamma_\mu (1 - \gamma_5) c | \Lambda_c(v) \rangle = \bar{u}_\Lambda \left(F_1^{\Lambda_c \Lambda}(v \cdot p) + \not{v} F_2^{\Lambda_c \Lambda}(v \cdot p) \right) \gamma_\mu (1 - \gamma_5) u_{\Lambda_c}. \quad (31)$$

Assuming a dipole q^2 behavior for form factors, the ratio $R = \tilde{F}_1^{\Lambda_c \Lambda} / \tilde{F}_2^{\Lambda_c \Lambda}$ is measured by CLEO to be [9]

$$R = -0.25 \pm 0.14 \pm 0.08. \quad (32)$$

The form factors $\tilde{F}_{1,2}$ are related to f 's and g 's by

$$f_1 = g_1 = \tilde{F}_1 + \frac{m_f}{m_i} \tilde{F}_2, \quad f_2 = f_3 = g_2 = g_3 = \frac{\tilde{F}_2}{m_i}. \quad (33)$$

Since R is independent of q^2 if \tilde{F}_1 and \tilde{F}_2 have the same q^2 dependence, we can apply the quark model (22) to get

$$\tilde{F}_1(q_m^2) = 1 + \frac{\bar{\Lambda}}{4m_s}, \quad \tilde{F}_2(q_m^2) = -\frac{\bar{\Lambda}}{4m_s}, \quad (34)$$

which lead to

$$R = - \left(1 + \frac{4m_s}{\bar{\Lambda}} \right)^{-1} = -0.23 \quad (35)$$

for $m_s = 510$ MeV. This is in excellent agreement with experiment (32), but it should be stressed that $1/m_c$ corrections, which are potentially important, have not been included in (32) and (35).

III. Applications

In this section we will apply the baryonic form factors obtained in the nonrelativistic quark model to various physical processes. Since the calculation of the q^2 dependence of form factors is beyond the scope of a nonrelativistic quark model, we will thus assume a pole dominance for the form-factor q^2 behavior:

$$f(q^2) = \frac{f(0)}{\left(1 - \frac{q^2}{m_V^2}\right)^n}, \quad g(q^2) = \frac{g(0)}{\left(1 - \frac{q^2}{m_A^2}\right)^n}, \quad (36)$$

where m_V (m_A) is the pole mass of the vector (axial-vector) meson with the same quantum number as the current under consideration. In practice, either monopole ($n = 1$) or dipole ($n = 2$) q^2 dependence are adopted in the literature. For definiteness, we will choose the dipole behavior suggested by the following argument. Considering the function

$$G(q^2) = \left(\frac{1 - q_m^2/m_*^2}{1 - q^2/m_*^2} \right)^n, \quad (37)$$

with m_* being the pole mass, it is clear that $G(q^2)$ plays the role of the baryonic Isgur-Wise function $\zeta(v \cdot v')$ in $\Lambda_Q \rightarrow \Lambda_{Q'}$ transition, namely $G = 1$ at $q^2 = q_m^2$. The function $\zeta(\omega)$ has been calculated in two different models:

$$\zeta(\omega) = \begin{cases} 0.99 \exp[-1.3(\omega - 1)], & \text{soliton model [10];} \\ \left(\frac{2}{\omega+1}\right)^{3.5+\frac{1.2}{\omega}}, & \text{MIT bag model [11].} \end{cases} \quad (38)$$

Using the pole masses $m_V = 6.34$ GeV and $m_A = 6.73$ GeV for the transition $\Lambda_b \rightarrow \Lambda_c$, we find that $G(q^2)$ is compatible with $\zeta(\omega)$ only if $n = 2$. However, one should bear in mind that, in reality, the q^2 behavior of form factors is probably more complicated and it is likely that a simple pole dominance only applies to a certain q^2 region.

Before proceeding to applications we would like to make a remark on the role played by heavy quark symmetry here. Though HQET is employed in Sec. II as a benchmark for testing if the nonrelativistic quark model calculations of form factors are reliable and trustworthy, heavy quark symmetry is no longer relevant in all applications described in this section except for in Eq.(65) where we apply the static heavy quark limit to relate the tensor matrix element to the vector and axial-vector one. For example, the different but realistic vector and axial-vector pole masses used in (36) break heavy quark symmetry explicitly.

3.1 Semileptonic decay

We shall study in this subsection the decay rate for the semileptonic transition $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + e + \bar{\nu}_e$. Take the semileptonic decay $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$ as an example. Since $\eta = 1$, $m_V = m_{D_s(1-)} = 2.11$ GeV, $m_A = m_{D_s(1+)} = 2.536$ GeV [7], the form factors at $q^2 = 0$ obtained from (22) and (36) with $n = 2$ are

$$\begin{aligned} f_1^{\Lambda_c \Lambda}(0) &= 0.50 N_{\Lambda_c \Lambda}, & f_2^{\Lambda_c \Lambda}(0) &= -0.25 N_{\Lambda_c \Lambda} / m_{\Lambda_c}, & f_3^{\Lambda_c \Lambda}(0) &= -0.05 N_{\Lambda_c \Lambda} / m_{\Lambda_c}, \\ g_1^{\Lambda_c \Lambda}(0) &= 0.65 N_{\Lambda_c \Lambda}, & g_2^{\Lambda_c \Lambda}(0) &= -0.06 N_{\Lambda_c \Lambda} / m_{\Lambda_c}, & g_3^{\Lambda_c \Lambda}(0) &= -0.32 N_{\Lambda_c \Lambda} / m_{\Lambda_c}, \end{aligned} \quad (39)$$

where uses of $m_c = 1.6$ GeV and $m_s = 510$ MeV have been made. From the flavor-spin wave function of Λ_c and Λ with a positive helicity along the z direction

$$\begin{aligned} |\Lambda_c \uparrow\rangle_{\text{flavor-spin}} &= \frac{1}{\sqrt{2}}(ud - du)c\chi_A, \\ |\Lambda \uparrow\rangle_{\text{flavor-spin}} &= \frac{1}{\sqrt{6}}[(ud - du)s\chi_A + (13) + (23)], \end{aligned} \quad (40)$$

where (ij) means permutation for the quark in place i with the quark in place j , we get

$$N_{\Lambda_c \Lambda} = {}_{\text{flavor-spin}} \langle \Lambda \uparrow | b_s^\dagger b_c | \Lambda_c \uparrow \rangle_{\text{flavor-spin}} = \frac{1}{\sqrt{3}}. \quad (41)$$

The computation of the baryon semileptonic decay rate is straightforward; for an analytic expression of the decay rate, see for example Ref.[12]. We obtain

$$\Gamma(\Lambda_c \rightarrow \Lambda e^+ \nu_e) = (N_{\Lambda_c \Lambda})^2 \times 2.11 \times 10^{11} s^{-1} = 7.1 \times 10^{10} s^{-1}, \quad (42)$$

which is in excellent agreement with experiment [7]

$$\Gamma(\Lambda_c \rightarrow \Lambda e^+ \nu_e)_{\text{expt}} = (7.0 \pm 2.5) \times 10^{10} s^{-1}. \quad (43)$$

Table I. Nonrelativistic quark model predictions for baryonic form factors evaluated at $q^2 = 0$ using dipole q^2 dependence and $|V_{cb}| = 0.040$ [15] (m_i being the mass of the parent heavy baryon). For a comparsion, we also present the nonrelativistic quark model predictions given in Refs.[5] and [6]. The numerical values of the former reference are quoted from Table IV of [5], while the predicted values of the latter are computed from Eqs.(3.48)-(3.51) of [6]. Pole masses are taken to be $m_V = 2.11$ GeV, $m_A = 2.536$ GeV for $B_c \rightarrow B$ transition and $m_V = 6.34$ GeV, $m_A = 6.73$ GeV for $B_b \rightarrow B_c$. Also shown are the spin and flavor factors for various baryonic transitions.

transition	η	N_{fi}	$f_1(0)$	$f_2(0)m_i$	$f_3(0)m_i$	$g_1(0)$	$g_2(0)m_i$	$g_3(0)m_i$	
$\Lambda_c^+ \rightarrow \Lambda^0$	1	$\frac{1}{\sqrt{3}}$	0.29	-0.14	-0.03	0.38	-0.03	-0.19	this work
	1	1	0.35	-0.09	0.25	0.61	0.04	-0.10	[5]
	1	$\frac{1}{\sqrt{3}}$	0.36	-0.17	-0.17	0.47	-0.22	-0.22	[6]
$\Xi_c^0 \rightarrow \Xi^-$	1	$\frac{1}{\sqrt{3}}$	0.31	-0.19	-0.04	0.39	-0.06	-0.24	this work
	1	1	0.48	-0.08	0.26	0.76	0.04	-0.12	[5]
	1	$\frac{1}{\sqrt{3}}$	0.40	-0.23	-0.23	0.50	-0.30	-0.30	[6]
$\Lambda_b^0 \rightarrow \Lambda_c^+$	1	1	0.53	-0.12	-0.02	0.58	-0.02	-0.13	this work
	1	1	0.60	-0.14	-0.14	0.63	-0.15	-0.15	[6]
$\Xi_b^0 \rightarrow \Xi_c^+$	1	1	0.54	-0.14	-0.02	0.58	-0.03	-0.16	this work
	1	1	0.62	-0.17	-0.17	0.67	-0.18	-0.18	[6]
$\Omega_b^- \rightarrow \Omega_c^0$	$-\frac{1}{3}$	1	0.72	0.68	-0.36	-0.20	0.01	0.06	this work
	$-\frac{1}{3}$	1	0.85	0.81	-0.60	-0.23	0.08	0.08	[6]

It must be stressed that the flavor factor $N_{\Lambda_c \Lambda} = 1/\sqrt{3}$, which was already noticed in [6,13] and particularly accentuated by [14], is very crucial for an agreement between theory and experiment. In the literature it is customary to replace the s quark in the baryon Λ by the heavy quark Q to obtain the wave function of the Λ_Q . However, this amounts to assuming SU(4) or SU(5) flavor symmetry. Since SU(N)-flavor symmetry with $N > 3$ is badly broken, the flavor factor $N_{\Lambda_Q \Lambda}$ is no longer unity (of course, $N_{\Lambda_Q \Lambda_{Q'}} = 1$). Indeed, if

$N_{\Lambda_c \Lambda}$ were equal to one, the predicted rate for $\Lambda_c \rightarrow \Lambda e^+ \nu_e$ would have been too large by a factor of 3 !

For completeness, the numerical values of form factors at $q^2 = 0$ are tabulated in Table I and the nonrelativistic quark model predictions for the decay rates of semileptonic decays of heavy baryons are summarized in Table II. We will not consider the case of sextet heavy baryons as they are dominated by strong or electromagnetic decays (except for Ω_Q). Two remarks are in order. (i) We see from Tables I and II that the predictions of Ref.[5] for form factors and semileptonic decay rates are in general different substantially from ours. First, the important suppression factor of N_{fi} for antitriplet heavy baryon-octet baryon transition is not taken into account in [5]. Second, the calculated heavy-heavy baryon form factors in [5] at zero recoil do not satisfy the constraints imposed by heavy quark symmetry. The results of Ref.[6] are more close to ours. However, $1/m_Q$ corrections are not included in [6] and the computed $1/m_q$ effects there disagree with HQET for $\Lambda_b \rightarrow \Lambda_c$ and $\Xi_b \rightarrow \Xi_c$ transitions. (ii) The parameter $\bar{\Lambda}$ is process dependent; for example, it can be as large as 1.11 GeV for $\Omega_b \rightarrow \Omega_c e \bar{\nu}_e$, whereas it is only 0.61 GeV for $\Lambda_c \rightarrow \Lambda e^+ \nu_e$.

Table II. Nonrelativistic quark model predictions for the semileptonic decay rates in units of $10^{10} s^{-1}$ evaluated using dipole q^2 dependence for form factors. Values in parentheses are the predicted rates with QCD corrections. However, as stressed in [14], it seems that QCD effects computed in [5] are unrealistically too large.

process	[5]	[6]	this work	experiment [7]
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	18.0 (11.2)	9.8	7.1	7.0 ± 2.5
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	28.8 (18.1)	8.5	7.4	-
$\Lambda_b^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}_e$	-	5.9	5.1	-
$\Xi_b^0 \rightarrow \Xi_c^+ e^- \bar{\nu}_e$	-	7.2	5.3	-
$\Omega_b^- \rightarrow \Omega_c^0 e^- \bar{\nu}_e$	-	5.4	2.3	-

3.2 Nonleptonic decay

At the quark level, the nonleptonic weak decays of the baryon usually receive contributions from external W -emission, internal W -emission and W -exchange diagrams. At the hadronic level, these contributions manifest as factorizable and pole diagrams. It is known that, contrary to the meson case, the nonspectator W -exchange effects in charmed baryon decays are of comparable importance as the spectator diagrams [16]. Unfortunately, in general it is difficult to estimate the pole diagrams. Nevertheless, there exist some decay modes of heavy baryons which proceed only through the internal or external W -emission diagram.

Examples are

$$\begin{aligned} \text{internal } W\text{-emission} & : \Lambda_b \rightarrow J/\psi\Lambda, \quad \Xi_b \rightarrow J/\psi\Xi, \quad \Omega_b \rightarrow J/\psi\Omega, \quad \Lambda_c \rightarrow p\phi, \dots \\ \text{external } W\text{-emission} & : \Omega_b \rightarrow \Omega_c\pi, \quad \Omega_c \rightarrow \Omega\pi. \end{aligned} \quad (44)$$

Consequently, the above decay modes are free of nonspectator effects and their theoretical calculations are relatively clean.

In this subsection we shall study two of the decay modes displayed in (44), namely $\Lambda_b \rightarrow J/\psi\Lambda$ and $\Lambda_c \rightarrow p\phi$. The general amplitude of $\Lambda_b \rightarrow J/\psi\Lambda$ has the form

$$A(\Lambda_b \rightarrow J/\psi\Lambda) = i\bar{u}_\Lambda(p_\Lambda)\varepsilon^{*\mu}[A_1\gamma_\mu\gamma_5 + A_2(p_\Lambda)_\mu\gamma_5 + B_1\gamma_\mu + B_2(p_\Lambda)_\mu]u_{\Lambda_b}(p_{\Lambda_b}), \quad (45)$$

where ε_μ is the polarization vector of the J/ψ . Under factorization assumption, the internal W -emission contribution reads

$$A(\Lambda_b \rightarrow J/\psi\Lambda) = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*a_2\langle J/\psi|\bar{c}\gamma_\mu(1-\gamma_5)c|0\rangle\langle\Lambda|\bar{s}\gamma^\mu(1-\gamma_5)b|\Lambda_b\rangle, \quad (46)$$

where a_2 is an unknown parameter introduced in Ref.[17]. It follows from (45) and (46) that

$$\begin{aligned} A_1 &= -\lambda[g_1^{\Lambda_b\Lambda}(m_{J/\psi}^2) + g_2^{\Lambda_b\Lambda}(m_{J/\psi}^2)(m_{\Lambda_b} - m_\Lambda)], \\ A_2 &= -2\lambda g_2^{\Lambda_b\Lambda}(m_{J/\psi}^2), \\ B_1 &= \lambda[f_1^{\Lambda_b\Lambda}(m_{J/\psi}^2) - f_2^{\Lambda_b\Lambda}(m_{J/\psi}^2)(m_{\Lambda_b} + m_\Lambda)], \\ B_2 &= 2\lambda f_2^{\Lambda_b\Lambda}(m_{J/\psi}^2), \end{aligned} \quad (47)$$

with $\lambda = \frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*f_{J/\psi}m_{J/\psi}$. Since $\eta = 1$, $N_{\Lambda_b\Lambda} = \frac{1}{\sqrt{3}}$, $m_V = m_{B_s(1^-)} \cong 5.42$ GeV and $m_A = m_{B_s(1^+)} \cong 5.86$ GeV, we find from Eqs.(22) and (36) that

$$\begin{aligned} f_1^{\Lambda_b\Lambda}(m_{J/\psi}^2) &= 0.131, & f_2^{\Lambda_b\Lambda}(m_{J/\psi}^2) &= -0.054/m_{\Lambda_b}, \\ g_1^{\Lambda_b\Lambda}(m_{J/\psi}^2) &= 0.203, & g_2^{\Lambda_b\Lambda}(m_{J/\psi}^2) &= -0.036/m_{\Lambda_b}. \end{aligned} \quad (48)$$

The decay rate reads [18]

$$\Gamma(\Lambda_b \rightarrow J/\psi\Lambda) = \frac{p_c}{8\pi} \frac{E_{J/\psi} + m_{J/\psi}}{m_{\Lambda_b}} \left[2(|S|^2 + |P_2|^2) + \frac{E_{J/\psi}^2}{m_{J/\psi}^2}(|S + D|^2 + |P_1|^2) \right], \quad (49)$$

with the S , P and D waves given by

$$\begin{aligned} S &= -A_1, \\ P_1 &= -\frac{p_c}{E_{J/\psi}} \left(\frac{m_{\Lambda_b} + m_\Lambda}{E_\Lambda + m_\Lambda} B_1 + m_{\Lambda_b} B_2 \right), \\ P_2 &= \frac{p_c}{E_\Lambda + m_\Lambda} B_1, \\ D &= -\frac{p_c^2}{E_{J/\psi}(E_\Lambda + m_\Lambda)} (A_1 - m_{\Lambda_b} A_2), \end{aligned} \quad (50)$$

where p_c is the c.m. momentum. Using $|V_{cb}| = 0.040$ [15], $\tau(\Lambda_b) = 1.07 \times 10^{-12} \text{s}$ [7], $a_2 \sim 0.23$ [19], and $f_{J/\psi} = 395 \text{ MeV}$ extracted from the observed $J/\psi \rightarrow e^+e^-$ rate, $\Gamma(J/\psi \rightarrow e^+e^-) = (5.27 \pm 0.37) \text{ keV}$ [7], we find

$$\mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) = 2.1 \times 10^{-4}. \quad (51)$$

When anisotropy in angular distribution is produced in a polarized Λ_b decay, it is governed by the asymmetry parameter α given by [18]

$$\alpha = \frac{4m_{J/\psi}^2 \text{Re}(S^* P_2) + 2E_{J/\psi}^2 \text{Re}(S + D)^* P_1}{2m_{J/\psi}^2(|S|^2 + |P_2|^2) + E_{J/\psi}^2(|S + D|^2 + |P_1|^2)}. \quad (52)$$

Numerically, it reads ⁵

$$\alpha(\Lambda_b \rightarrow J/\psi \Lambda) = -0.11, \quad (53)$$

where the negative sign of α reflects the $V - A$ structure of the current.

The $\Lambda_b \rightarrow J/\psi \Lambda$ decay was originally reported by the UA1 Collaboration [21] with the result

$$F(\Lambda_b) \mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) = (1.8 \pm 0.6 \pm 0.9) \times 10^{-3}, \quad (54)$$

where $F(\Lambda_b)$ is the fraction of b quarks fragmenting into Λ_b . Assuming $F(\Lambda_b) = 10\%$ [21], this leads to

$$\mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) = (1.8 \pm 1.1)\%. \quad (55)$$

However, both CDF [22] and LEP [23] did not see any evidence for this decay. For example, based on the signal claimed by UA1, CDF should have reconstructed 30 ± 23 $\Lambda_b \rightarrow J/\psi \Lambda$ events. Instead CDF found not more than 2 events and concluded that

$$F(\Lambda_b) \mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) < 0.50 \times 10^{-3}. \quad (56)$$

The limit set by OPAL is [23]

$$F(\Lambda_b) \mathcal{B}(\Lambda_b \rightarrow J/\psi \Lambda) < 1.1 \times 10^{-3}. \quad (57)$$

Hence, a theoretical study of this decay mode would be quite helpful to clarify the issue. The prediction (51) indicates that the branching ratio we obtained is two orders of magnitude smaller than what expected from UA1 (55).

⁵Our previous study on $\Lambda_b \rightarrow J/\psi \Lambda$ [20] has a vital sign error in Eq.(14) for the expression of the D -wave amplitude, which affects the magnitude of the decay rate and the sign of decay asymmetry. Moreover, the important flavor factor $N_{\Lambda_b \Lambda} = 1/\sqrt{3}$ is not taken into account there.

We next turn to the Cabibbo-suppressed decay $\Lambda_c \rightarrow p\phi$. As emphasized in Ref.[16], this decay mode is of particular interest because it provides a direct test of the large- N_c approach in the charmed baryon sector, though this approach is known to work well for the nonleptonic weak decays of charmed mesons. From the flavor-spin wave function of the Λ_c (40) and the proton

$$|p \uparrow\rangle_{\text{flavor-spin}} = \frac{1}{\sqrt{3}}[uud\chi_s + (13) + (23)], \quad (58)$$

we get

$$N_{\Lambda_c p} = \frac{1}{\sqrt{2}}. \quad (59)$$

Since the calculation is very similar to that of $\Lambda_b \rightarrow J/\psi\Lambda$, we simply write down the results:

$$\mathcal{B}(\Lambda_c \rightarrow p\phi) = (c_2)^2 2.26 \times 10^{-3}, \quad \alpha(\Lambda_c \rightarrow p\phi) = -0.10, \quad (60)$$

where we have applied $f_\phi = 237$ MeV, $m_{D(1-)} = 2.01$ GeV and $m_{D(1+)} = 2.42$ GeV. As the Wilson coefficient c_2 is expected to be of order -0.56 in the large- N_c approach, it follows that

$$\mathcal{B}(\Lambda_c \rightarrow p\phi) = 7.1 \times 10^{-4}. \quad (61)$$

Therefore, in order to test $1/N_c$ expansion and the nonrelativistic quark model for the form factors, the experimental accuracy should be reached at the level of a few 10^{-4} . Experimentally, the branching ratio is measured to be

$$\mathcal{B}(\Lambda_c \rightarrow p\phi) = \begin{cases} (1.8 \pm 1.2) \times 10^{-3}, & \text{ACCMOR [24];} \\ < 1.7 \times 10^{-3}, & \text{E687 [25].} \end{cases} \quad (62)$$

Finally, it is worth remarking that it is important to take into account the effect of the flavor-suppression factor (e.g. $N_{\Lambda_c\Lambda}$) on the factorizable contributions to the nonleptonic two-body decays of charmed baryons; such effects thus far have not been considered in the literature [16,26].

3.3 Weak radiative decay

Recently the weak radiative decays of B mesons and bottom baryons have been systematically studied in Ref.[27]. At the quark level, there are two essential mechanisms responsible for weak radiative decays: electromagnetic penguin mechanism and W -exchange (or W -annihilation) bremsstrahlung. The two-body decays of the bottom baryons proceeding through the short-distance electromagnetic penguin diagrams are:

$$\Lambda_b^0 \rightarrow \Sigma^0 \gamma, \quad \Lambda^0 \gamma, \quad \Xi_b^0 \rightarrow \Xi^0 \gamma, \quad \Xi_b^- \rightarrow \Xi^- \gamma, \quad \Omega_b^- \rightarrow \Omega^- \gamma. \quad (63)$$

In this subsection, we shall study the above weak radiative decay modes using the nonrelativistic quark model in conjunction with the heavy b -quark symmetry.

To begin with, the electromagnetic penguin-induced radiative decay amplitude is [27]

$$A(B_i \rightarrow B_f + \gamma) = i \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2(x_t) V_{tb} V_{ts}^* m_b \varepsilon^\mu q^\nu \times \langle B_f | \bar{s} \sigma_{\mu\nu} [(1 + \gamma_5) + \frac{m_s}{m_b} (1 - \gamma_5)] b | B_i \rangle, \quad (64)$$

where q is the photon momentum, F_2 is a smooth function of $x_t \equiv m_t^2/M_W^2$ [28] and it is numerically equal to 0.65 for $\Lambda_{\text{QCD}} = 200$ MeV and $m_t = 174$ GeV. In order to evaluate the tensor matrix elements in (64), we consider the static heavy b -quark limit so that

$$\langle B_f | \bar{s} i \sigma_{0i} (1 + \gamma_5) b | B_i \rangle = \langle B_f | \bar{s} \gamma_i (1 - \gamma_5) b | B_i \rangle. \quad (65)$$

Hence,

$$\begin{aligned} \langle B_f | \bar{s} i \sigma_{0i} (1 + \gamma_5) b | B_i \rangle \varepsilon^0 q^i &= \frac{1}{2} \langle B_f | \bar{s} \gamma_i (1 - \gamma_5) b | B_i \rangle (\varepsilon^0 q^i - \varepsilon^i q^0) \\ &= \bar{u}_f i \sigma_{0i} \varepsilon^0 q^i [f_1 - f_2(m_i + m_f) + g_1 \gamma_5 + g_2(m_i - m_f) \gamma_5] u_i. \end{aligned} \quad (66)$$

It follows from (64) and (66) that

$$A(B_i \rightarrow B_f + \gamma) = i \bar{u}_f (a + b \gamma_5) \sigma_{\mu\nu} \varepsilon^\mu q^\nu u_i, \quad (67)$$

with

$$\begin{aligned} a &= \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2(x_t) m_b V_{tb} V_{ts}^* [f_1(0) - f_2(0)(m_i + m_f)], \\ b &= \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2(x_t) m_b V_{tb} V_{ts}^* [g_1(0) + g_2(0)(m_i - m_f)], \end{aligned} \quad (68)$$

being parity-conserving and -violating amplitudes, respectively. The decay rate is

$$\Gamma(B_i \rightarrow B_f + \gamma) = \frac{1}{8\pi} \left(\frac{m_i^2 - m_f^2}{m_i} \right)^3 (|a|^2 + |b|^2). \quad (69)$$

In order to apply the heavy quark symmetry relation (65), we shall neglect $1/m_b$ corrections to the form factors given in (22). To the leading order in $1/m_b$, we obtain

$$\begin{aligned} \Gamma(\Lambda_b \rightarrow \Lambda \gamma) &= 1.6 \times 10^{-18} \text{ GeV}, \\ \Gamma(\Xi_b \rightarrow \Xi \gamma) &= 2.2 \times 10^{-18} \text{ GeV}, \end{aligned} \quad (70)$$

and a prohibited $\Lambda_b \rightarrow \Sigma \gamma$, where uses of $N_{\Lambda_b \Lambda} = N_{\Xi_b \Xi} = 1/\sqrt{3}$ and $V_{tb} V_{ts}^* \approx -V_{cb} V_{cs}^*$ have been made. Therefore,

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) = 2.7 \times 10^{-6}, \quad (71)$$

for $\tau(\Lambda_b) = 1.07 \times 10^{-12}s$ [7]. In Ref.[27] two different methods, namely the heavy s -quark approach and the MIT bag model, have been employed to estimate the decay rate of $\Lambda_b \rightarrow \Lambda \gamma$. Our present result (71) is somewhat smaller than the prediction given in [27] owing to the presence of the flavor-suppression factor of $1/\sqrt{3}$ in the amplitude.

IV. Conclusions and discussion

In the heavy quark effect theory (HQET), current-induced $1/m_Q$ corrections and the presence of higher dimensional operators in the effective Lagrangian are the two sources of $1/m_Q$ effects on the hadronic form factors. Since the predictive power of HQET for baryon form factors at order $1/m_Q$ is limited only to antitriplet-antitriplet heavy baryon transition, this motivates us to apply the nonrelativistic quark model to evaluate the weak current-induced baryonic form factors at zero recoil in the rest frame of the heavy parent baryon, where the quark model is most trustworthy. Contrary to previous similar work, we have shown that the HQET predictions for antitriplet-antitriplet heavy baryon transitions at $v \cdot v' = 1$ are reproducible in the nonrelativistic quark model. Moreover, the latter approach has two eminent features. First, it becomes meaningful to consider $1/m_s$ corrections so long as the recoil momentum is smaller than the m_s scale. Second, $1/m_Q$ effects arising from wave-function modifications vanish at zero recoil in the quark model. Consequently, the nonrelativistic quark model results for the form factors evaluated at maximum q^2 are applicable to any heavy-heavy and heavy-light baryonic transitions.

An obvious criterion for testing the reliability of quark model calculations is that model results must satisfy all the constraints imposed by heavy quark symmetry. In the heavy quark limit, normalizations of heavy-heavy form factors and hence some relations between form factors at zero recoil are fixed by heavy quark symmetry. These constraints are not respected in Ref.[5]. While this discrepancy is improved in the work of [6], its prediction for $\Lambda_b \rightarrow \Lambda_c$ (or $\Xi_b \rightarrow \Xi_c$) form factors at order $1/m_Q$ is still too large by a factor of 2 when compared with HQET. We have shown that our prescription of quark model calculations does incorporate the features of heavy quark symmetry (a careful examination on the underlying assumptions we have made is discussed in Sec.II) and hence can be applied to compute baryon form factors beyond the arena of HQET.

As the conventional practice, we make the pole dominance assumption for the q^2 dependence to extrapolate the form factors from maximum q^2 to the desired q^2 point. We argued that a dipole q^2 behavior is more preferred since it is close to the baryonic Isgur-Wise function calculated recently. Nevertheless, one should bear in mind that the assumption of pole dominance for form factors is probably too simplified and this problem remains unresolved.

We have applied our main results (22) in this paper to various decays of heavy baryons. In all model applications described in Sec.III, heavy quark symmetry is no longer relevant except for in Eq.(65). It turns out that the inclusion of a flavor suppression factor, which occurs in most of heavy to light baryonic transitions, is very crucial to explain the experimental observation of the semileptonic decay $\Lambda_c \rightarrow \Lambda e^+ \nu_e$. The presence of this flavor suppression factor, which is missed in most literature, will of course affect the predictions on the decay rates of many decay modes involving a transition from heavy to light baryons. It is conceivable that some of our predictions can be tested soon in the near future. Examples are $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$, $\Lambda_b \rightarrow \Lambda e^- \bar{\nu}_e$, $\Lambda_b \rightarrow J/\psi \Lambda$, $\Lambda_c \rightarrow p \phi$ and $\Lambda_b \rightarrow \Lambda \gamma$. A particularly interesting decay mode is the channel $\Lambda_b \rightarrow J/\psi \Lambda$. Its branching ratio is predicted to be 2×10^{-4} , which is two orders of magnitude smaller than the UA1 observation but consistent with the limit set by CDF and LEP.

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